MEASURING OF CONDITIONAL VALUE AT RISK PORTFOLIO USING COPULA

Krisada Khruachalee[1,*] and Winai Bodhisuwan[2]

Abstract

The uncertainty of return on investment is a major concern for the vast majority of investors. Under normal market conditions, a portfolio's risk exposure over a specific time frame with a predetermined confidence level can be measured. Since a portfolio's return is rarely characterized by the assumption of a multivariate normal distribution, the use of normality Value-at-Risk (VaR) plays a crucial role in risk mitigation, but can generate an undesirable measure of risk exposure for portfolio investment. With a dynamic tool in modeling multivariate distribution regardless of the assumption of joint normality, applying a copula is a practical alternative choice for extracting a cumulative joint distribution for a portfolio's return. The applications in this work are illustrated by the portfolios of the four largest and the four smallest market capitalization stocks in the tourism and hospitality sector. It was found that the portfolio returns of the large and small market capitalization portfolios were characterized by logistic and Student's *t* distributions respectively. Consequently, the VaR and conditional VaR based on the Gaussian copula, could be used to characterize and estimated the distributions of the respective portfolio returns according to the logistic and Student's *t* distributions. The conditional VaR of the large and small market capitalization portfolios calculated from the copula method provides a slightly higher level of risk than the conditional VaR and the VaR with the assumption of a multivariate normal distribution. Moreover, the small market capitalization portfolio provides slightly higher values of VaR and CVaR than the large market capitalization portfolio for all assumptions of VaR. Therefore, the use of conditional VaR

[1,*] Mr. Krisada Khruachalee obtains two master's degrees - Master of Science (Finance) from Faculty of Commerce and Accountancy, Chulalongkorn University, and Master of Science (Applied Statistics) from National Institute of Development Administration (NIDA). Currently, he is doing his Ph.D. in Statistics at the Department of Statistics, Faculty of Science, Kasetsart University and also working as a full-time lecturer in the Department of Finance and Banking, Faculty of Business Administration, Siam Technology College. Email: krisada.khr@ku.th

[2] Assoc. Prof. Dr. Winai Bodhisuwan obtains a Ph.D. in Operational Research from Royal Melbourne Institute of Technology, Australia. Currently, he is working as a full-time lecturer in the Department of Statistics, Faculty of Science, Kasetsart University, Thailand.

based on the Gaussian copula is more reasonable for investors who conservatively manage their investment portfolios. However, managing the investment portfolio based on a conservative level does not completely imply the performance of portfolio management. On the other hand, an accurate value of VaR, directly estimated from the actual distribution of a portfolio's returns, provides a vital means of assessing better portfolio management. Due to being sensitively volatile to several surrounding factors within the hospitality and tourism sector, implementing a conservative investment strategy is more suitable.

Keywords: Conditional Value-at-Risk, Multivariate Distribution Function, Copula, Tourism & Hospitality

INTRODUCTION

The stock exchange is an important investment channel for investors to allocate their resources in risky assets according to their risk appetite which reflects their expectation to receive a favourable return in the future.

However, the assets sold in the stock exchange always contain a large swing in their value when information reflects the investors' expectations. In the last quarter of the year 2019, the Thailand stock index (SET) continuously declined when the outbreak of the 2019 Coronavirus originated in Wuhan, spreading widely, and causing more patients to become sick or to die from the disease. The index sharply dropped to a more than three-year low in line with a sharp jump in infections outside mainland China, and increases in new coronavirus infections in Thailand, strongly undermining confidence in the Thai economy. Not only the devaluation of the Thai stock index, but also the inevitable slowdown of the hospitality and tourism industry, one of the most important sectors in generating income for Thailand, led to deterioration of investors' confidence. This situation was exacerbated as Chinese tourists are considered as the most significant nationality regarding visits to Thailand, accounting for the largest proportion of tourism income.

Due to the COVID-19 pandemic, the government declared a state of emergency and imposed strict travel bans for foreign tourists coming to Thailand. As a result, foreign tourist arrivals completely collapsed. According to the data collected by the Ministry of Tourism and Sports (2020), the service sector GDP contracted by 1.1%, mainly from declines in the number of tourists, negatively affecting tourism and tourism-related sectors, particularly accommodation and food service activities. In addition, the tourism industry is expected to suffer more than a \$50 billion loss in revenue or about 9.5% of GDP for 2020. Many informal businesses are at risk, depending on tourists' spending, which fell 40% in the first quarter of 2020. The COVID-19 pandemic is considered as a source of systematic risk, seriously affecting global stock markets with uncertainty and resulting in a large swing of share prices. Nhamo et al., (2020) observed that the news of the COVID-19 outbreak and related measures included travel bans, and bans on mass gatherings, put in place to curb the spread of the disease, dampening the stock markets and leading to declines in tourism-related stock prices. Therefore, tourism firms and those in their value chain became the worst performers on global stock markets.

It is unavoidable that investors who had invested in tourism and hospitality stock must assess the potential risks that may arise from their investments. Managing an investment is considered as risk management where some investors passively accept risks, while other investors may intentionally attempt to create competitive strategies to eliminate their risk exposure. However, risk is an important factor for both corporations and investors, especially for the financial industry who must carefully monitor this factor due to its damaging effects. There are two significant risk exposures considered by a corporation where business risk is regarded as a business decision companies make and the business environment in which they operate (Jorion, 2007). In addition, broad macroeconomic risks are those included in the business environment. Furthermore, the potential losses unsettled in financial market activities are classified as a financial risk where financial managers carefully monitor various kinds of risks such as liquidity risk, credit risk, operational risk, and market risk. Many researchers and professional risk managers intensively discover the procedure for identifying, measuring, and managing financial risks to mitigating collapse. Jorion (2007) proposed one possible course of action in setting-up stop-loss limits, in which the cumulative loss cannot exceed a certain limit. However, there is no assurance to confirm that the loss will closely match the pre-determined limit. Another approach for risk measurement is the concept of duration which firstly solves the assets' price given the current yield.

The next approach is to perform a sensitivity analysis that linearly measures the exposure of an asset's value to the yield. A further approach is to perform a scenario analysis or stress test which recalculates the portfolio's price over a range of yields. Unfortunately, these approaches are inadequate as they do not consider the volatility of the risk factors and the correlation which could diversify across the market. With the limitations of the conventional risk measurement methods, Value at Risk (VaR) was proposed to combine the relationship between price and yield with the probability of a hostile market movement. Thereby, the VaR is significantly considered as a statistical risk measure of potential losses which accounts for both correlation and leverage. VaR has been extensively applied to measure liquidity risk, credit risk, operational risk, and market risk. Nonetheless, there is nothing new about the idea of VaR as it draws from the mean-variance framework developed by Markowitz (1952).

Investment in the stock market is generally the holding of many company's equities at the same time as a portfolio with various investment objectives through strategic and tactical asset allocation. For instance, Balcilar et al. (2015) studied the total risk exposure of stock investment in 10 various industries composed in the Islamic Sector Indexes. These objectives oblige an equity portfolio to meet a certain predetermined condition of risk tolerance. The risk tolerance and actual risk level of a portfolio often uses VaR as the risk measurement in which the value of VaR may be specified in monetary terms or as a percentage of the investment value at the beginning of the investment period. There are several works illustrating the benefits of VaR and its applications in risk measurement, such as Jorion (2002), Yamai and Yoshiba (2005), Allen et al. (2012a), Allen et al. (2012b) and Dargiri et al. (2013).

There are still some arguments regarding the use of VaR considering the issue that the calculation does not produce true results, therefore increasing the risk and associated damages higher than the acceptable level. In addition, having multiple models resulted in different risk values, making it more difficult to choose a suitable model to apply (Marshall and Seigel, 1997). Nevertheless, this risk can be reduced by testing the accuracy of the model via the method of backtesting.

Since the VaR is used to restrict the risk but the results may not be satisfactory, another method called Conditional Value at Risk (CVaR) or Expected Shortfall was initiated by Artzner et al. (1997). The advantage of the CVaR over VaR is having the ability to measure the benefits of diversification. The disadvantage of CVaR is that its calculations are more complex and difficult to understand. Based on the four properties of acceptable risk feature proposed by Artzner et al. (1999), the VaR does not have the property of sub-additive in which the investment diversification to multiple securities causes an increase of VaR since the VaR gives the weight to calculate only the interested quantile. On the other hand, CVaR provides the same weight for all information exceeding the amount of the interesting quantile or beyond the confidence level. Rockafellar and Uryasev (2002) found that using CVaR provides the ability to measure risk that is greater than the normal risk value (VaR). This result is also consistent with Yamai and Yoshiba (2005).

Due to global economic crises, there have been several improvements in investment theory, especially the Modern Portfolio Theory initiated by Markowitz (1952). This theory proposed crucial assumptions for a portfolio’s return, where there is normality and time is invariant. It can be implied that expected returns are linearly correlated to each other. Unfortunately, there are also numerous pieces of evidence proving that daily returns of stock do not lie within the normal distribution. Therefore, the traditional portfolio theory is inescapably suspected when the portfolio's risk factors are measured according to normality, which can lead to underestimating the value at risk (VaR). For instance, the evidence found by Longin and Solnik (2001) and Ang and Chen (2002) shows that the return of assets is more highly correlated during the course of market downturns when the risk of a portfolio could be higher than expected.

With a dynamic tool in modelling multivariate distribution, regardless of the assumption of joint normality, applying a copula is a more practical alternative option in which the copula provides a multivariate joint distribution merging the marginal distribution and the dependence between the variables. Conversely, the copula can decompose any d-dimensional joint distribution into d marginal distributions and a copula function. In addition, Salvadori et al. (2007) supported that the further dominance of copula is the easiest to apply with various complex marginal distributions such as finite mixtures which increasingly draw the attention of researchers. It may be broadly found that the copula has been generally used in financial applications where the papers of Bouyé et al. (2000), Embrechts et al. (2002) and Embrechts et al. (2003) are general examples of copula used to model risk limits and extreme values.

Additionally, the papers of Cherubini and Luciano (2001) and Fortin and Kuzmics (2002) also applied the copula to estimate VaR in different aspects. Cherubini and Luciano (2001) used the Archimedean copula family and the historical empirical distribution to estimate the marginal distribution. Fortin and Kuzmics (2002) used a linear combination of copula to estimate the portfolio's VaR which composed of the FSTE and DAX stock indices.

In this work, we discuss the concepts and applications of the copula in measuring a conditional value-at-risk (CVaR) portfolio. This paper is organized as follows. The various estimation approaches for portfolio value-at-risk and a complementary measure (conditional value-at-risk) are first defined. Sklar's theorem and the concept of the copula is then presented. The estimation approach and model selection criteria for copula's family is also discussed. Finally, the proposed risk measurement is applied to measure two different stock portfolios, where the first portfolio is composed of the four largest tourism and hospitality market capitalization companies listed in the Stock Exchange of Thailand, namely Central Plaza Hotel PCL. (CENTEL), Shangri-La Hotel PCL. (SHANG), Dusit Thani PCL. (DTC), and The Erawan Group PCL. (ERW). The second portfolio is composed of the four smallest tourism and hospitality market capitalization companies listed in the Stock Exchange of Thailand which are Veranda Resort PCL. (VRANDA), Asia Hotel PCL. (ASIA), City Sports and Recreation PCL. (CSR), and the Mandarin Hotel PCL. (MANRIN). For both the large and small capitalization tourism and hospitality stock portfolios, the contributions of CVaR using copula in modelling multivariate distribution are empirically examined.

RESEARCH OBJECTIVES

With a dynamic tool in modelling multivariate distributions, this work aims to present the concepts and properties of copula function as well as an application of the copula in estimation of the conditional value at risk (CVaR) of the two stock portfolios, where the first portfolio is composed of the four largest market capitalization stocks listed in the tourism and hospitality sector of the Stock Exchange of Thailand. The second portfolio is composed of the four smallest tourism and hospitality market capitalization stocks listed on the Stock Exchange of Thailand during February 1, 2010, to November 22, 2019.

MATHEMATICAL DEFINITION OF VALUE AT RISK

There are diverse approaches to estimate the VaR depending on the requirements of the probability distribution, such as the historical simulation approach, Monte Carlo approach, and the analytical or variance-covariance approach. The historical simulation approach does not require knowledge of any probability distribution of the asset returns, as the value of VaR can be measured based on the sample's quantile. On the other hand, the Monte Carlo and the analytical approaches require knowledge of the probability distribution of the asset's returns where the VaR is derived from the standard deviation.

Historical Simulation Approach of Value at Risk

Sorting the portfolio's return (R^p) in ascending order, the VaR is the smallest value of the portfolio's return in which the percentile at the level of confidence c is located.

$$VaR = R_c^p$$

where R_c^p is the percentile of a portfolio's return at the level of confidence c.

Analytical Approach of Value at Risk

Since VaR is a statistical measurement of the worst loss depending on the current position, the predetermined probability distribution that a certain loss will be larger than the VaR is defined as:

$$P\left(L > VaR\right) \leq 1 - c$$

where c denotes the confidence level and L denotes the loss level.

With a known probability distribution for the portfolio's

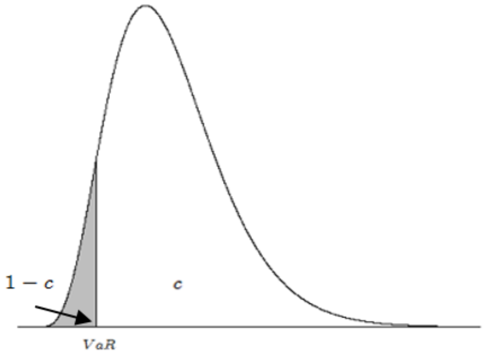

Figure 1: Graphical illustration of Value at Risk

return (R_p), in general form the VaR can be derived from the probability distribution of the portfolio's return, where R_p^* represents the worst possible loss realization of the investment portfolio. In other words, R_p^* is the quantile of the probability distribution where the cut-off value is predetermined by the probability of being exceeded.

$$P(R_p \le R_p^*) = \int_{-\infty}^{R_p^*} f(R_p)\,dR_p = 1-c$$

With the portfolio investment theory developed by Markowitz (1952), the portfolio's return (R_p) is described by the weighted average of all individual expected returns held in the portfolio, where w_i denotes the percentage composition of a particular holding of assets in a portfolio and R_i denotes the expected rate of return for each individual asset.

$$R_p = w_1R_1 + w_2R_2 + \cdots + w_NR_N = \sum_{i=1}^{N} w_iR_i$$

To shorten the notation of the portfolio's return, this can be written in matrix notation as:

$$R_p = w'R = \begin{bmatrix} w_1 & w_2 & \cdots & w_N \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_N \end{bmatrix}$$

where w' denotes the transposed vector of weights given to each asset. and R denotes the vertical vector of individual assets' return.

Consequently, the portfolio's expected return ($E(R_p)$) is:

$$E(R_p) = \mu_p = w'\mu = \sum_{i=1}^{N} w_i\mu_i$$

and the portfolio's variance ($V(R_p)$) is:

$$V(R_p) = \sigma_p^2 = \sum_{i=1}^{N} w_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} w_iw_j\sigma_{ij}$$
$$= \sum_{i=1}^{N} w_i^2\sigma_i^2 + 2\sum_{i=1}^{N}\sum_{j>i}^{N} w_iw_j\sigma_{ij}$$

If there are a number of assets in the portfolio, the portfolio's variance

$(V(R_p))$ can be more conveniently written in matrix notation as:

$$\sigma_p^2 = w'\Sigma w$$

$$= \begin{bmatrix} w_1 \cdots w_N \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_N^2 \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

where Σ denotes the variance-covariance matrix of the portfolio's return.

To estimate the exposure X in monetary terms, the portfolio's variance is multiplied by the initial portfolio's investment value W.

$$\sigma_p^2 W^2 = X'\Sigma X$$

The portfolio's variance has thus far been related with the distribution of the portfolio's return, in order to estimate the VaR. Assuming that $F(R_1, R_2, \ldots, R_N)$ and $f(R_1, R_2, \ldots, R_N)$ are the cumulative density function (cdf) and the probability density function (pdf) of a joint random variable of the assets' returns $(R_1, R_2, \ldots, R_N)$ respectively, the risk's level is $VaR(R_p)$ with c confidence level in terms of the percentage of the investment value at the beginning of the investment period, and can be derived by multiplying R (resulting from the following equation) by -1.

$$\int_{-\infty}^{R} \cdots \int_{-\infty}^{R - w_N R_N \ldots - w_2 R_2} f(R_1, .., R_N) dR_1 .. dR_N$$

$$= 1 - c$$

The valuation of the VaR based on the previous equation is practically problematic. However, when evaluating the risk level by the VaR for a short time horizon such as 1 day, it can generally be assumed that the joint probability distribution of $(R_1, R_2, \ldots, R_N)$ is normal, in which the vector of the expected value is zero and the variance-covariance matrix is Σ.

For the traditional portfolio theory, the returns of all individual securities are assumed to be normally distributed. Therefore, the confidence level c, can simply be transformed into a standard normal deviate α. Therefore, the probability of losing more than $-\alpha$ is $1-c$.

$$VaR_p = \alpha \sigma_p W = \alpha \sqrt{X'\Sigma X}$$

The portfolio's variance basically depends on variances, covariances, and the number of assets in the portfolio. However, the scale of covariance depends on the variances of the individual assets which are not easy to interpret. Thus, the correlation coefficient (ρ_{ij}) is proposed to overcome this complication as the ρ_{ij} is a scale-free measure of linear dependence which is represented by:

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$$

To extend the ability of the VaR in measuring the portfolio's risk where there are a number of risky assets in the portfolio, an alternative complementary measure called conditional value at risk, expected shortfall, conditional loss, or expected tail loss (ETL) can provide more information on how much could be lost if we blow beyond the VaR.

$$CVaR = E(R_p \mid R_p < VaR)$$

$$= \frac{\int_{-\infty}^{VaR} R_p f(R_p) dR_p}{\int_{-\infty}^{VaR} f(R_p) dR_p}$$

In case of a standard normal variate, the CVaR can be shortened to:

$$E(R_p \mid R_p < -\alpha) = \frac{-\Phi(\alpha)}{F(-\alpha)}$$

where Φ denotes the standard normal cumulative distribution function.

Skla's Theorem and Copula

This section provides the definition of copula and an equitable definition for the context of the random variable where the copula has been used to describe the dependence structure between random variables.

Suppose the marginals of a random vector $\left(X_1, X_2, \ldots, X_d\right)$ are continuous, whereby the marginal cdf is $F_i(x_i) = P(X_i \le x_i)$. With the integration of each component, the random vector
$(U_1, \ldots, U_d) = (F_1(X_1), \ldots, F_d(X_d))$
has uniformly distributed marginals.

The copula of $\left(X_1, X_2, \ldots, X_d\right)$ is defined as the joint cumulative distribution function of $(U_1, U_2, \ldots, U_d)$.

$$C(u_1, \ldots, u_d) = P(U_1 \le u_1, \ldots, U_d \le u_d)$$

The benefit of this expression is to generate a pseudo-random sample from the multivariate probability distribution. The required sample can be illustrated as:
$\left(X_1, \ldots, X_d\right) = (F_1^{-1}(U_1), \ldots, F_d^{-1}(U_d))$

Since F_i is assumed to be continuous, the inversion of F_i^{-1} is uncomplicated and can be revised as:
$C(u_1, \ldots, u_d)$
$= P(X_1 \le F_1^{-1}(u_1), \ldots, X_d \le F_d^{-1}(u_d))$

Definition 1. A d-dimensional copula is a function C, whose domains are $\left[0,1\right]^d$ and whose range is $\left[0,1\right]$ with the following properties:

1. $C(u_1, \ldots, u_{i-1}, 0, u_{i+1}, \ldots, u_d) = 0$, when at least one element of u is 0;
2. $C(1, \ldots, 1, u, 1 \ldots, 1) = u$, if one element is u and all others are 1.
3. For instance, $C: \left[0,1\right]^2 \to \left[0,1\right]$ is a bivariate copula if $C(x) = 0$ for all $x \in \left[0,1\right]^2$ when at least one element of x is 0.

In addition, $C(x_1, 1) = C(1, x_2) = 1$ for all $(x_1, x_2) \in \left[0,1\right]^2$.

Moreover, for all (a_1, a_2), $(b_1, b_2) \in \left[0,1\right]^2$ with $a_1 \le b_1$ and $a_2 \le b_2$, we have:

$$V_C([a,b]) = C(a_2,b_2) - C(a_1,b_2) - C(a_2,b_1) + C(a_1,b_1)$$

where $V_C([a,b]) \geq 0$

Definition 2. In the bivariate case, the copula function C is the joint distribution function of the random vector $U = (U_1, U_2)^t$ where $U_i = F_i(X_i)$ and F_i is the marginal distribution function of X_i, $i = 1,2$ which can be illustrated as follows:

$$H(x_1,x_2) = C(F_1(x_1), F_2(x_2))$$

where H is the joint distribution of (X_1, X_2).

Consequently, it can be assumed that a copula is any bivariate distribution function whose marginal distributions are a standard uniform distribution.

Sklar's Theorem

Theorem 1. Every multivariate cumulative distribution function of a random vector $(X_1, X_2, \ldots, X_d)$ can be expressed using Sklar's theorem in terms of its marginal distribution and a copula C as follows:

$$H(x_1,\ldots,x_d) = P[X_1 \leq x_1, \ldots, X_d \leq x_d] = C(F_1(x_1), F_2(x_2), \ldots, F_d(x_d))$$

If this multivariate distribution has a density h, the relationship between the pdf of asset returns and the copula pdf can be assumed to be:

$$h(x_1,..,x_d) = c(F_1(x_1),..,F_d(x_d)) \cdot f_1(x_1) \cdot\cdot f_d(x_d)$$

where c is the copula's density.

Theorem 2. In the bivariate case, if F_1 and F_2 are continuous, the copula C is unique on $Ran(F_1) \times Ran(F_2)$.

Conversely, for all $x \in \overline{\Re}^n$, if C is a copula and F_1, F_2 are distribution functions, then the function H is:

$$H(x_1,x_2) = C(F_1(x_1), F_2(x_2))$$

where H is a joint distribution function with marginal distributions F_1 and F_2. The derivation of these expressions can be found in Nelsen (2007).

THE ESTIMATION OF CONDITIONAL VALUE AT RISK USING COPULA

Fitting the Distributions of Stock Return

In determining the marginal distribution of each stock return, the Anderson-Darling test was employed to evaluate the possible distributions that could best describe the behaviour of the stock return. The probability distributions considered for comparison purposes consisted of the normal distribution, Student's *t*-distribution, log-normal distribution, logistic distribution, triangular distribution, generalized beta distribution, and generalized extreme value distribution. The stationary time series data of daily historical returns from before the outbreak of the 2019 Coronavirus (February 1, 2010, to November 22, 2019), was used for analysis of the hospitality and tourism companies connected with the four largest and the four smallest market capitalization stocks registered in the Stock Exchange of Thailand. It was found that the logistic distribution provides an outstanding description of the returns of all four of the largest market capitalization stocks. In addition, it was found that the Student's *t*-distribution best describes the returns for all four of the smallest market capitalization stocks. Even though the normal distribution does not provide a reasonable fit with these data sets, the logistic and Student's *t* distributions are implicitly similar in shape to the normal distribution (i.e. bell shaped) because they are symmetrical and unimodal. However, the tailed distribution of the logistic and Student's t are slightly fatter than those of the normal distribution.

Considering the portfolio of the four largest market capitalization stocks, it was assumed that the return R_i is a logistic distribution whose domain is in the range of $(-\infty,\infty)$. The distribution is determined by two parameters (α and β). The location parameter α explains where the distribution is centered on the horizontal axis. The scale parameter β explains what the spread of the distribution is. The probability density function (pdf) and cumulative density function (cdf) of the logistic distribution are calculated respectively as follows:

$$f(R_i)=\frac{\exp\left\{-\dfrac{R_i-\alpha}{\beta}\right\}}{\beta\left(1+\exp\left\{-\dfrac{R_i-\alpha}{\beta}\right\}\right)^2},$$

$$F(R_i)=\frac{1}{1+\exp\left\{-\dfrac{R_i-\alpha}{\beta}\right\}}$$

where the mean and variance of R_i are α and $\dfrac{\beta^2\pi^2}{3}$ respectively.

For the portfolio of the four smallest market capitalization stocks, it was assumed that the return R_i fits the Student's *t*-distribution whose domain also lies in the range of $(-\infty,\infty)$. However, the characteristic of the Student's *t*-distribution is determined by a positive integer shape parameter which is the degree of freedom (ν) that went into the estimate of the standard deviation. With greater degrees of freedom, the Student's *t*- distribution is almost indistinguishable from the normal distribution. The probability density function (pdf) and cumulative density function (cdf) of the Student's *t*-distribution are calculated respectively as follows:

$$f(R_i) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{R_i^2}{\nu}\right)^{\frac{-(\nu+1)}{2}}$$

$$F(R_i) = \frac{1}{2} + R_i\Gamma\left(\frac{\nu+1}{2}\right)\frac{{}_2F_1\left(\frac{1}{2},\frac{\nu+1}{2};\frac{3}{2};-\frac{R_i^2}{\nu}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)}$$

where Γ is the Gamma function and ${}_2F_1$ is the hypergeometric function.

Due to the computational complexity of the pdf and cdf of both distributions, the R programming language was employed to estimate the parameters of each distribution. Moreover, various packages of the R programming language were applied for various purposes.

With the package "*fitdistrplus*" proposed by Delignette-Muller and Dutang (2015), the parameters of each probability distribution which best describe each stock return can be estimated by applying a maximum likelihood estimation method. The estimated parameters of the logistic ($\hat{\alpha}$ and $\hat{\beta}$) and Student's t ($\hat{\nu}$) distributions are shown in Table 1 and Table 2 respectively. In addition, the *"plotdist"* function of Delignette-Muller and Dutang (2015) was applied to provide the plots of empirical and theoretical density for each daily stock return as illustrated in Figure 2-9.

Since the logistic distribution has no shape parameter, the logistic pdf thus has only one shape which is the bell shape. As illustrated in Figure 2-5, it was found that the shape of the distribution does not change but the pdf of CENTEL and SHANG were shifted to the right. On the other hand, the pdf of DTC and ERW were shifted to the left. Obviously, it can be seen that the shape of the logistic distribution is very similar to that of the normal distribution. The main difference lies in the tails of the distribution.

Table 1: The estimated parameters $\hat{\alpha}$ and $\hat{\beta}$ which determine the location and scale of the logistic distribution of each stock return in the large market capitalization portfolio.

Symbol	Estimated Parameters	
	Alpha ($\hat{\alpha}$)	Beta ($\hat{\beta}$)
CENTEL	0.05105	1.17904
SHANG	0.02484	1.18389
DTC	-0.01939	1.00443
ERW	-0.00854	1.06660

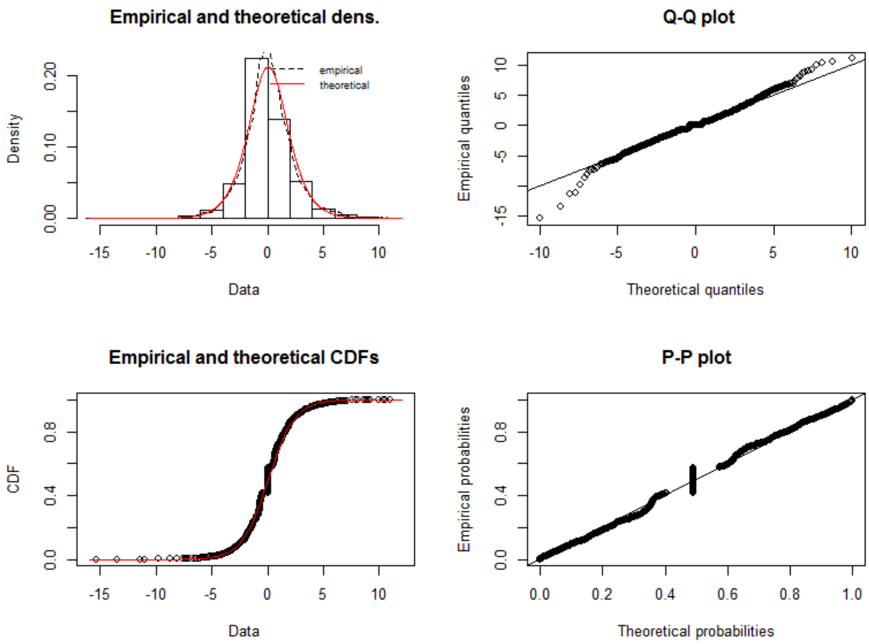

Figure 2: The plots of empirical and theoretical density for the daily return of CENTEL

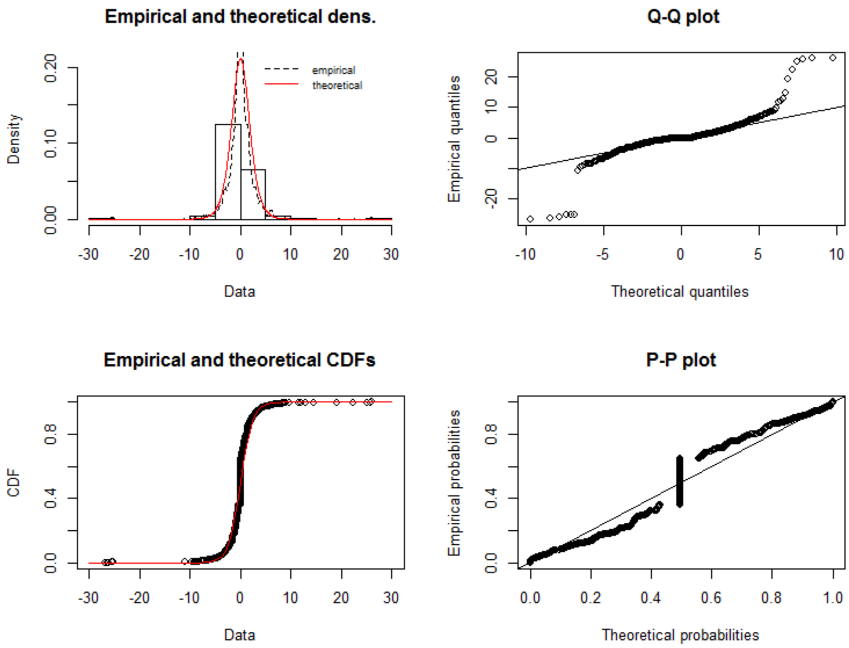

Figure 3: The plots of empirical and theoretical density for the daily return of SHANG

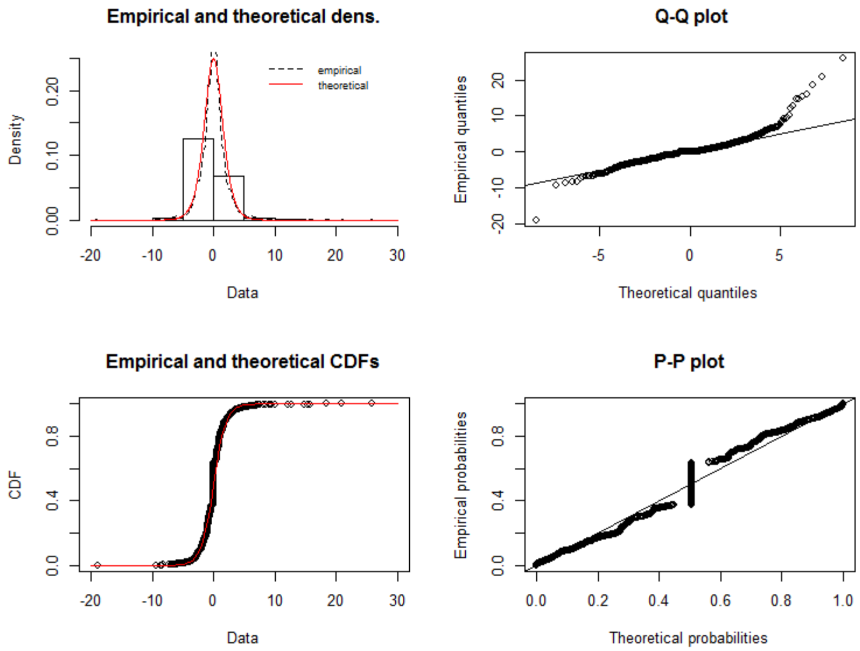

Figure 4: The plots of empirical and theoretical density for the daily return of DTC

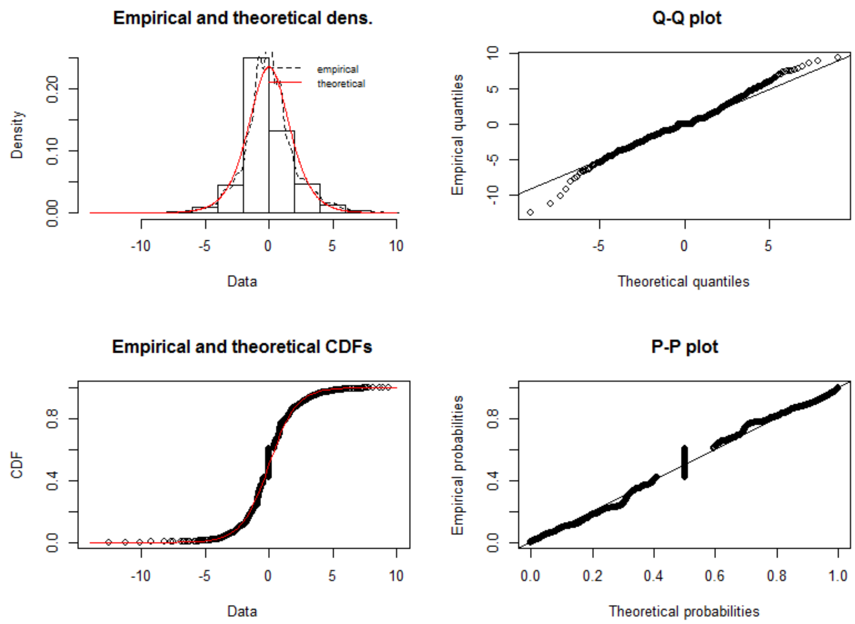

Figure 5: The plots of empirical and theoretical density for the daily return of ERW

The shape of the distribution for each stock return in the small market capitalization portfolio is indistinguishably the same, as the estimated degree of freedom ($\hat{\nu}$) for each distribution is very similar. Even when the degree of freedom increases, the Student's *t*-distribution is more favorable than the normal distribution as it provides the lowest score of the Akaike information criterion (AIC) for the model comparison.

Table 2: The estimated parameter $\hat{\nu}$ which determines the shape of the Student's *t*- distribution for each stock return in the small market capitalization portfolio.

Symbol	Estimated Parameters
	Degree of freedom ($\hat{\nu}$)
VRANDA	294.4942
ASIA	294.6770
CSR	298.6299
MANRIN	297.5352

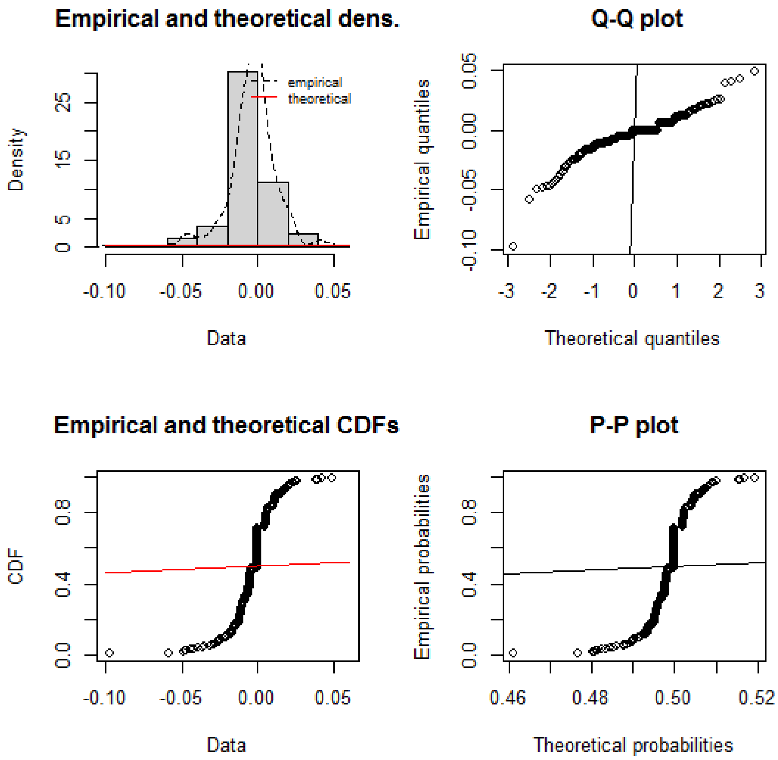

Figure 6: The plots of empirical and theoretical density for the daily return of VRANDA

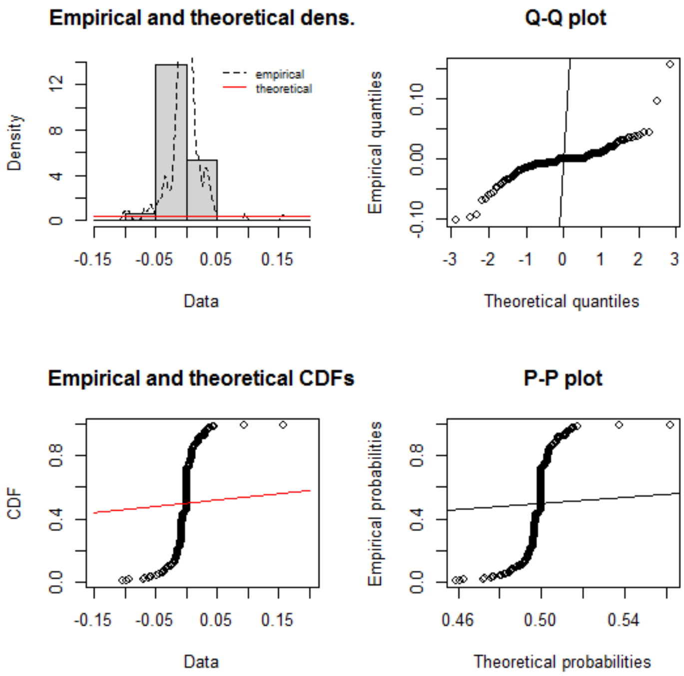

Figure 7: The plots of empirical and theoretical density for the daily return of ASIA

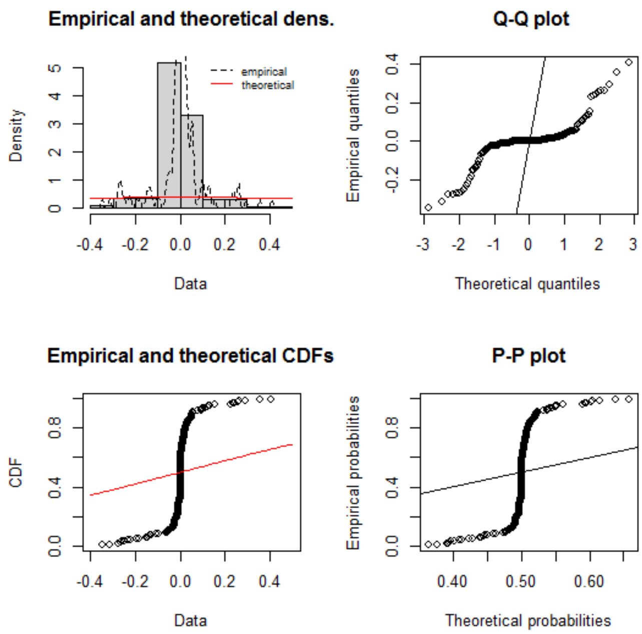

Figure 8: The plots of empirical and theoretical density for the daily return of CSR

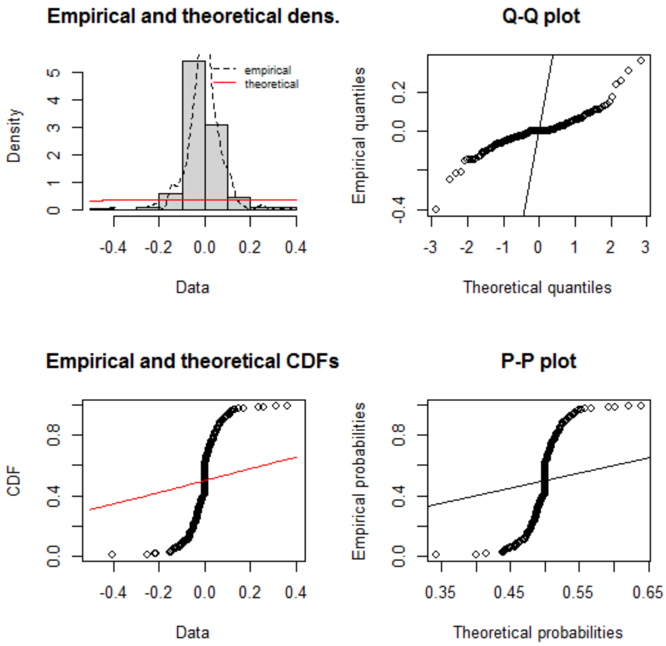

Figure 9: The plots of empirical and theoretical density for the daily return of MANRIN

As the returns for each stock in the large market capitalization portfolio are distributed logistically, this matches the theory of Johnson and Kotz (1972), which suggests that the joint distribution of the stock return ($R_1,\ldots,R_N$) is also the joint logistic distribution, whereby the joint pdf and joint cdf can be shown respectively as:

$$f(R_1,\ldots,R_N) = N!\left[1+\sum_{i=1}^{N}\exp\left\{\frac{R_i-\alpha_i}{\sigma_i}\right\}\right]^{-(N+1)} \times(-1)\sum_{i=1}^{N}\exp\left\{\frac{R_i-\alpha_i}{\sigma_i}\right\},$$

and

$$F(R_1,\ldots,R_N) = \frac{1}{1+\sum_{i=1}^{N}\exp\left\{-\frac{R_i-\alpha_i}{\sigma_i}\right\}}$$

where $\sigma_i = \sqrt{\frac{\beta_i^2\pi^2}{3}}$.

Since the Student's *t*-distribution provides the best description for each stock return in the small market capitalization portfolio, this follows the theory of Kotz and Nadarajah (2004) which suggests that the joint distribution of the stock return (R) is also the joint Student's *t*-distribution whereby the joint pdf can be shown as:

$$f(R)=\frac{\Gamma\left(\frac{\nu+P}{2}\right)}{\sqrt{(\nu\pi)^{P}}\,\Gamma\left(\frac{\nu}{2}\right)}$$

$$\times\left[1+\frac{1}{\nu}(R-\mu)^{T}\Sigma^{-1}(R-\mu)\right]^{-\frac{(\nu+P)}{2}}$$

where a random vector $R=(R_1,\ldots,R_P)$ has the $P-$variate Student's *t*-distribution with the degree of freedom ν,
μ denotes the mean vector,
and Σ denotes the covariance matrix.

The Simulation Approach for Conditional VaR using Copula

Since the determination of the joint probability distribution function is practically problematic, the estimation of the VaR can be alternatively evaluated by the copula joint distribution function (copula cdf) and the joint density function (copula pdf) as substitute functions. Therefore, the conditional VaR can be evaluated based on the pseudo-random samples generated by the copula technique.

While the joint distribution of assets' returned in the portfolio can be described by the distribution function $F(R_1,R_2,\ldots,R_N)$ and the copular function, in the form of $C(F_1(R_1),\ldots,F_N(R_N))$, the distribution of the portfolio's return $R_p=w_1R_1+\ldots+w_NR_N$ can then be reasonably described by the functions of $F(\cdot)$ and $C(\cdot)$.

Assuming that $f_p(R_p)$ is the pdf of the portfolio's return R_p, if R_p is randomly selected from a large number of random variables, applying the joint copula distribution function, the estimation of conditional VaR is then straightforward as follows:

Step 1: Randomly select N random variables (Z) from a standard normal distribution in which each variable is also identically distributed. In addition, each variable also shares its relationship through the variance-covariance matrix §.

With the "*mvrnorm*" function of Venables and Ripley (2002) in the R package "MASS" produced by the R CORE TEAM. (2020), it is straightforward to generate random samples from a multivariate normal distribution where the customizable variance-covariance matrix (§) can be smoothly fitted to the observed data.

Step 2: Calculate the vector of cumulative probability (u) based on the random sample Z with the function "*pnorm*" of Venables and Ripley (2002) in the R package, where the transformation does not alter the variance-covariance structure among the random variables. It was consequently found that each distribution of the new random variables contained in u was uniformly distributed in the $[0,1]$ interval.

Step 3: It was found that the natural financial data have heavier tails than in an observed normal distribution. Therefore, the tail behaviour of each copula such as the Student-*t* copula which is similar to the Gaussian distribution but has positive tail dependence should be applied (Daniel, 2016). However, another parameter, the degrees of freedom (ν) must also be estimated. As the value of ν increases, the Student-*t* distribution becomes closer to the Gaussian distribution. The Gaussian copula, which is tail independent and also allows for negative dependence then becomes widely applied in many fields of finance and risk management. Li (2000) proposed that the transformation of random variables contained in u to become the return of stock R_i can be performed through the Gaussian copula as follows:

$$C(u_1,\ldots,u_N;\rho) = \Phi_\rho(\Phi^{-1}(u_1),\ldots,\Phi^{-1}(u_N))$$

where $u_i = F_i(R_i)$

Φ_ρ is a normal cumulative joint distribution function of a multivariate Gaussian distribution with a mean vector zero and $n \times n$ correlation matrix ρ.

Φ^{-1} is a probability density function whose values are inversed from a normal cumulative distribution.

ρ is the correlation matrix between variable $\Phi^{-1}(R_i)$ and $\Phi^{-1}(R_j)$.

The return of each stock R_i in the large market capitalization portfolio, matched with the element of $\Phi(u_i)$ at the i^{th} of $\Phi_\rho(U)$, and can be express by:

$$R_i = \hat{\alpha}_i - \hat{\beta}_i - \ln\left\{\frac{1}{\Phi(U_i)} - 1\right\}$$

where $\hat{\alpha}_i$ is the estimated location parameter of stock return i where a logistic distribution is assumed.

$\hat{\beta}_i$ is the estimated scale parameter of stock return i where a logistic distribution is assumed.

Since the quantile function of the Student's *t*-distribution is an intractable case, the return of each stock R_i for the small market capitalization portfolio which is matched with the element of $\Phi(u_i)$ at the i^{th} of $\Phi_\rho(U)$ will be solved by the "*qt*" function of the R core team (2020) in the R programming language.

Step 4: Calculate the portfolio's return (R_p) based on the weighted average of all individual expected returns held in the portfolio. Each stock return (R_i) generated from **Step 3** was weighted with the respective investment portion. In this paper, it was assumed that the weight (w_i) given to each stock return (R_i) was equally distributed ($w_i = 0.25$), resulting with the portfolio return (R_p).

Step 5: Recalculate a portfolio return (R_p) based on **Step 1** to **Step 4** M number of times. In this paper, the portfolio return was calculated with M = 50,000. Therefore, there are 50,000 values of the portfolio's return (R_p).

Step 6: Sort the 50,000 values of the portfolio returns (R_p) in ascending order. The portfolio's VaR is then gathered by multiplying the portfolio's return (R_p) by -1, where it is located at the c 100 quantiles.

Step 7: In order to estimate the complementary measure, conditional VaR or expected tail loss (ETL) is another risk indicator widely used for risk management. The conditional VaR provides a reasonable property of coherent risk measures in any conditions of the joint distribution. Artzner et al. (1997) proposed that the VaR is considered a good indicator of the risk level in terms of coherent risk measures only when the distribution of an asset's returns is classified as an elliptical family. However, the distributions of the portfolio's return (R_p) which comprised of the four largest market capitalization stocks and the four smallest market capitalization stocks in the hospitality and tourism sector were empirically examined according to the logistic and Student's *t* distributions which are not classified as part of the elliptical family. Therefore, the VaR with a non-elliptical distribution is not sufficient as a risk indicator. Artzner et al. (1999) examined the desirable properties of VaR, noting that it increases as a ratio of the investment amount (positive homogeneity). In addition, investing in risk-free assets reduces the value of VaR to the true value (transitional invariance). Implementing the diversification strategy may therefore increase the value of VaR rather than decreasing it (sub-additivity).

Since the VaR only gives weight to the interested quantile and ignores the weight given to the lower quantile, the VaR has many desirable properties except its sub-additivity. Accordingly, the conditional VaR, which provides the same weight to all data exceeds the interested quantile, and is a useful supplementary measure for risk indication, where the conditional VaR at c confidence level can be defined as:

$$Conditional\ VaR = -E\left[R_p \mid R_p < -VaR_c\right]$$

Once the value of the portfolio's return (R_p) has been obtained in ascending order from **Step 6**, the conditional VaR can be calculated by multiplying the average portfolio return, R_p (which is losses exceeding the negative VaR) by -1.

In order to compare the performance of the VaR and conditional VaR based on the Gaussian copula, where the probability distribution of the large market capitalization portfolio's return (R_p^L) is characterized by the logistic distribution and the probability distribution of the small market capitalization portfolio's return (R_p^S) is characterized by the Student's t-distribution, the VaR and conditional VaR were also computed under the conventional assumption that the portfolio's return follows a multivariate normal distribution as shown in Table 3 and Table 4.

As shown in Table 3 and Table 4, the results imply that the application of the VaR and conditional VaR based on the Gaussian copula for the large and small market capitalization stock portfolios in hospitality and tourism consistently lie in the same direction, where the conditional VaR based on the Gaussian copula is slightly higher than the normality conditional VaR, normality VaR, and Gaussian copula VaR for all given levels of confidence respectively. However, the small market capitalization portfolio provides slightly higher values of VaR and CVaR in comparison to the large market capitalization portfolio for all assumptions of VaR. It can be suggested that the small market capitalization portfolio in hospitality and tourism empirically has a larger chance of declining in asset value than the large market capitalization portfolio.

Even though, safe investments rarely significantly exceed the VaR, the application of CVaR in terms of risk exposure is considered safer than usual. Therefore, it can be implied that the conditional VaR based on the Gaussian copula is preferably applicable for a conservative portfolio investment where investors prioritize the preservation of capital by investing in lower-risk securities such as money market securities, blue-chip

Table 3: Risk comparison of the portfolio of the four largest market capitalization stocks in the hospitality and tourism industry based on various assumptions of VaR

Confidence Level	Normality		Gaussian Copula	
	VaR	Conditional VaR	VaR	Conditional VaR
99.0 %	1.2969%	1.3313%	1.2037%	1.5352%
97.5 %	1.1764%	1.1971%	0.9397%	1.2398%
95.0 %	0.9319%	0.9566%	0.8528%	0.9869%

Table 4: Risk comparison of the portfolio of the four smallest market capitalization stocks in the hospitality and tourism industry based on various assumptions of VaR

Confidence Level	Normality		Gaussian Copula	
	VaR	Conditional VaR	VaR	Conditional VaR
99.0 %	1.9319%	2.1135%	1.7731%	2.4625%
97.5 %	1.5443%	1.7642%	1.2986%	1.9863%
95.0 %	1.1672%	1.2893%	0.9354%	1.5396%

stock, and fixed income securities. Such investors will prefer to use the conditional VaR based on a Gaussian copula as a measurement of risk. Generally, investors are looking for a small CVaR. However, a large CVaR is also often found from investments with the most upside potential.

Nevertheless, managing portfolio investment based on a conservative level does not imply the best performance of portfolio management. On the other hand, better portfolio management requires an estimated value of VaR which provides the closest level to the true value which is directly estimated from the actual probability distribution of the portfolio's return (R_p). Due to its popularity and conceptual simplicity, the conditional VaR and VaR are still useful tools in providing a means of assessing how much risk exposure investors are taking in order to achieve their portfolio returns.

CONCLUSION

The research described in this article found that the portfolio returns for the four largest and the four smallest market capitalization stocks in the tourism and hospitality sector are respectively characterized by the logistic and Student's *t* distributions. Therefore, measuring values of VaR and conditional VaR for the portfolios with a multivariate normal distribution assumption on the portfolio returns may provide an undesirable value of risk level due to estimated errors which may arise.

With a dynamic tool in modelling multivariate distribution regardless of the assumption of joint normality distribution, the VaR and conditional VaR based on the Gaussian copula, where the distributions of the portfolio's returns are characterized by the logistic and Student's *t* distribution can then be used as an alternative measure to mitigate the risk level of the portfolio's return.

The conditional VaR calculated from the copula method provides a slightly higher level of risk than the conditional VaR and VaR with the assumption of the multivariate normal distribution for all given levels of confidence. However, the copula VaR provides the lowest value of risk level, aligning with the work of Khanthavit (2007) which studied the copula VaR for measuring the risk level of the Thai bond portfolio. The use of the conditional VaR based on the Gaussian copula is therefore more reasonable for investors who conservatively manage their portfolio than using the conditional VaR and VaR with the assumption of a multivariate normal distribution. Although, the major goal of conservative investment is to protect the principal of the portfolio, managing the portfolio based on a conservative level does not completely imply positive performance of portfolio management. On, the other hand, an accurately estimated value of the conditional VaR or VaR where they are directly estimated from the actual probability distribution of portfolio returns (R_p) provides a vital means of assessing better portfolio management.

Since the hospitality and tourism sector is sensitively volatile to several surrounding factors, implementing a conservative investment strategy is more suitable for portfolio investment during the COVID-19 pandemic. With these constructive results, the conditional VaR based on the Gaussian copula reasonably contributes a great benefit to investors who are focussed mainly on principal protection and those who carry the greatest chance of preserving the purchasing power of their capital with the least amount of risk.

REFERENCES

Allen, D. E., Powell, R. J., and Singh, A. K. (2012a). Beyond Reasonable Doubt: Multiple Tail Risk Measures Applied to European Industries, *Applied Economics Letters*, 19(7), 671-676.

Allen, D., Boffey, R., Kramadibrata, A., Powell, R., and Singh, A. (2012b). Thumbs Up to Parametric Measures of Relative VaR and CVaR in Indonesian Sectors, *International Journal of Business Studies*, 20(1), 27-42.

Ang, A., and Chen, J. (2002). Asymmetric Correlations of Equity Portfolios, *Journal of Financial Economics*, 63(3), 443-494.

Artzner, P., Delbaen, D., Eber, J., and Heath, D. (1997). Thinking Coherently, *Risk*, 10, 68-71.

Artzner, P., Delbaen, D., Eber, J., and Heath, D. (1999). Coherent Measures of Risk, *Mathematical Finance*, 9(3), 203-228.

Balcilar, M., Demirer, R., and Hammoudeh, S. (2015). Global Risk Exposures and Industry Diversification with Shariah-Compliant Equity Sectors, *Pacific-Basin Finance Journal*, 35(Part B), 499-520.

Bouye, E., Durrleman, V., Nikeghbali, A., Riboulet, G., and Roncalli, T. (2000). *Copulas for Finance, A Reading Guide and Some Applications*. Working Paper, Financial Econometrics Research Centre, City University, London.

Cherubini, U., and Luciano, E. (2001). Value-at-Risk Trade-off and Capital Allocation with Copulas. *Economic Notes*, 30(2), 235-256.

Daniel, J. (2016). Credit risk lecture notes. *MSc Mathematical Finance Lectures, Module* 8, April 2016.

Dargiri, M. N., Shamasabadi, H. A., Thim, C. K., Rasiah, D., and Sayedy, B. (2013). Value-at-Risk and Conditional Value-at-Risk Assessment and Accuracy Compliance in Dynamic of Malaysian Industries, *Journal of Applied Sciences*, 13(7), 974-983.

Delignette-Muller, M. L. and Dutang, C. (2015). *fitdistrplus: An R Package for Fitting*

Distributions. Journal of Statistical Software, 64(4), 1–34.

Embrechts, P., McNeil, A., and Straumann, D. (2002). *Correlation and Dependence in Risk Management: Properties and Pitfalls. Risk Management: Value at Risk and Beyond,* Cambridge University Press, 2002.

Embrechts, P., Hoing, A., and Juri, A. (2003). Using Copulae to Bound the Value-at-Risk for Functions of Dependent Risks*, Finance and Stochastics,* 7(2), 145-167.

Fortin, I., and Kuzmics, C. (2002). Tail-Dependence in Stock-Return Pairs. *International Journal of Intelligent Systems in Accounting, Finance & Management,* 11(2), 89-107.

Johnson, N. L., and Kotz, S. (1972). *Distributions in Statistics: Continuous Multivariate Distributions*, John Wiley and Sons, New York.

Jorion, P. (2002). How Informative Are Value-at-Risk Disclosures?. *The Accounting Review*, 77(4), 911-931.

Jorion, P. (2007). *Value at Risk: The New Benchmark for Managing Financial Risk*, 3rd ed, New York: McGraw-Hill.

Khanthavit, A. (2007). Technical Notes copula VaR and copula expected shortfall for measuring the exposure of the Thai bond portfolio. *Journal of Business Administration*, 30(113), 13-24.

Kotz, S., and Nadarajah, S. (2004). *Multivariate T-Distributions and Their Applications*, Cambridge University Press

Li, D. X., (2000). On default correlation: A copula function approach. *The Journal of Fixed Income*, 9(4), 43-54.

Longin, F., and Solnik, B. (2001). Extreme Correlation of International Equity Markets. *The Journal of Finance*, 56(2), 649-676.

Markowitz, H. (1952). Portfolio Selection, *The Journal of Finance,* 7(1), 77-91.

Marshall, C., and Siegel, M. (1997). Value-at-Risk: Implementing a Risk Measurement Standard. *The Journal of Derivatives*, 4(3), 91-110.

Ministry of Tourism and Sport (2020). *Tourism Statistics*. Retrieved April 2, 2021 from https://www.adb.org/sites/default/files/linked-documents/54177-001-sd-12.pdf

Nelsen, R. B. (2007). *Introduction to Copulas*, New York: Springer Science and Business Media.

Nhamo, G., Dube, K., and Chikodzi, D., (2020). *Counting the Cost of COVID-19 on the Global Tourism Industry* (pp. 297-318), Springer, Cham.

R CORE TEAM., (2020). *R: A language and environment for statistical computing. R Foundation for Statistical Computing*, Vienna, Austria. URL https://www.R-project.org/.

Rockafellar, R. T., and Uryasev, S. (2002). Conditional Value-at-Risk for General Loss Distributions. *Journal of*

Banking and Finance, 26(7), 1443-1471.

Salvadori, G., De Michele, C., Kottegoda, N. T., and Rosso, R. (2007). *Extremes in Nature: An Approach Using Copulas*, New York: Springer Science and Business Media.

Venables, W. N., and Ripley, B. D. (2002). *Modern Applied Statistics with S.* Fourth Edition., New York, Springer Science and Business Media.

Yamai, Y., and Yoshiba, T. (2005). Value-at-Risk versus Expected Shortfall: A Practical Perspective*. Journal of Banking & Finance,* 29(4), 997-1015, 2005